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TEMPERATURE FIELD OF A HALF-SPACE WITH A THERMALLY THIN COATING IN PULSE MODES OF HEAT EXCHANGE WITH THE ENVIRONMENT

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Characteristic features of the process of formation of the temperature field at the boundary of a solid body simulated by a homogeneous half-space with a "thermally thin" coating are studied by methods of mathematical modeling in realization of the pulsed modes of heat exchange with the environment.

An important place in the theory of heat conduction is occupied by investigations of the temperature fields in solid bodies, at the boundaries of which nonstationary modes of heat exchange with the environment that lead to a time variation of the heat-transfer coefficient are realized [1–31]. In particular, the necessity of accounting for the dependence of the heat-transfer coefficient on time arises in problems of heat transfer in the presence of high-temperature effects accompanied by destruction of the surface layers of a thermally loaded solid body. Such situations can lead not only to activation or deterioration of the conditions of heat exchange with the environment but also to the specific features of evolution of the temperature profile in the process of its formation [31].

Practical realization of nonstationary modes of heat exchange with the environment can also be associated with destruction of the coating of a solid body (a layer of thermoinsulating material deposited on a heat-insulated surface) in the process of intense surface heating of it [32–35]. Here it should be noted that use of analytical methods of solution of problems of the considered class with account for the functional dependence of the heat-transfer coefficient on time leads to the necessity of overcoming difficulties of a fundamental character [28].

The main aim of the conducted investigations was to study the characteristic features of the process of formation of the temperature field at the boundary of a solid body simulated by a half-space with a "thermally thin" coating in realization of the pulsed modes of heat exchange with the environment and in the presence of ideal thermal contact in the half-space–coating system.

The object of the investigations was the simplest one-dimensional mathematical model of the studied process

$$\frac{\partial\theta(\xi,\mathrm{Fo})}{\partial\mathrm{Fo}}=\frac{\partial^2\theta(\xi,\mathrm{Fo})}{\partial\xi^2}\,,\quad \mathrm{Fo}>0\,,\quad \xi>0\,;$$

$$\frac{\partial\theta_0(\xi,\mathrm{Fo})}{\partial\mathrm{Fo}}=a^2\,\frac{\partial^2\theta_0(\xi,\mathrm{Fo})}{\partial\xi^2}\,,\quad \mathrm{Fo}>0\,,\quad -h<\xi<0\,;$$

$$\theta(\xi,0)=0=\theta_0(\xi,0)\,,\quad \theta(0+0,\mathrm{Fo})=\theta_0(0-0,\mathrm{Fo})\,;$$

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$$\left.\frac{\partial\theta(\xi,\mathrm{Fo})}{\partial\xi}\right|_{\xi=0+0} = \Lambda \left.\frac{\partial\theta_0(\xi,\mathrm{Fo})}{\partial\xi}\right|_{\xi=0-0}, \quad \left.\frac{\partial\theta_0(\xi,\mathrm{Fo})}{\partial\xi}\right|_{\xi=-h+0} = \mathrm{Bi}(\mathrm{Fo})\left\{\theta_0(\xi,\mathrm{Fo})\big|_{\xi=-h+0} - 1\right\}, \tag{1}$$

where $\theta(\xi, \mathrm{Fo}) \in L^2[0, \infty]$, i.e., for each fixed value $\mathrm{Fo} \geq 0$ the function $\theta(\xi, \mathrm{Fo})$ is integrable with the square in the spatial variable $\xi \in [0, \infty)$:

$$\xi = \frac{x}{x_*}, \quad \mathrm{Fo} = \frac{\kappa t}{x_*^2}, \quad \theta = \frac{T - T_0}{T_{\mathrm{env}} - T_0}, \quad \theta_0 = \frac{T_{\mathrm{coat}} - T_0}{T_{\mathrm{env}} - T_0}, \quad a^2 = \frac{\kappa_{\mathrm{coat}}}{\kappa}, \quad \Lambda = \frac{\lambda_{\mathrm{coat}}}{\lambda}, \quad \mathrm{Bi} = \frac{\alpha}{\lambda_{\mathrm{coat}}} x_*, \quad h = \frac{l}{x_*};$$

$$\mathrm{Bi}(\mathrm{Fo}) = \sum_{k=0}^{N} H_k \left\{J(\mathrm{Fo} - \mathrm{Fo}^{(k)}) - J(\mathrm{Fo} - \mathrm{Fo}^{(k+1)})\right\},$$

$$0 < \mathrm{Fo}^{(0)} < \mathrm{Fo}^{(1)} < \ldots < \mathrm{Fo}^{(N)} < \mathrm{Fo}^{(N+1)} = +\infty, \quad N \in \{0, 1, \ldots\},$$

Here N, $\{H_k\}_{k=0}^N$, and $\{\mathrm{Fo}^{(k)}\}_{k=0}^N$ are the known constants and $J(\mathrm{Fo})$ is the unit function – the Heaviside function [36].

The assumption of the presence of a "thermally thin" coating allows one to realize the idea of "concentrated capacity" [27, 37], according to which the integral-mean temperature over the coating thickness can be taken equal to the temperature at its boundary, i.e.,

$$\langle\theta(\mathrm{Fo})\rangle \overset{\Delta}{=} \frac{1}{h}\int_{-h}^{0} \theta_0(\xi,\mathrm{Fo})\, d\xi = \theta_0(0-0,\mathrm{Fo}) = \theta_0(-h+0,\mathrm{Fo}).$$

Thus, the initial mathematical model (1) can be simplified and transformed to the following:

$$\frac{\partial\theta(\xi,\mathrm{Fo})}{\partial\mathrm{Fo}} = \frac{\partial^2\theta(\xi,\mathrm{Fo})}{\partial\xi^2}, \quad \mathrm{Fo} > 0, \quad \xi > 0;$$

$$\theta(\xi, 0) = 0; \tag{2}$$

$$\left.\frac{\partial\theta(\xi,\mathrm{Fo})}{\partial\xi}\right|_{\xi=0} = \gamma^{-1}\beta(\mathrm{Fo})\left\{\theta(\xi,\mathrm{Fo})\big|_{\xi=0} - 1\right\} + \gamma^{-1}\left.\frac{\partial\theta(\xi,\mathrm{Fo})}{\partial\mathrm{Fo}}\right|_{\xi=0},$$

where

$$\theta(\xi,\mathrm{Fo}) \in L^2[0,\infty); \quad \beta(\mathrm{Fo}) \overset{\Delta}{=} (a^2/h)\,\mathrm{Bi}(\mathrm{Fo}); \quad \gamma \overset{\Delta}{=} a^2/(\Lambda h);$$

$$\beta(\mathrm{Fo}) = \sum_{k=0}^{N} \beta_k \left\{J(\mathrm{Fo} - \mathrm{Fo}^{(k)}) - J(\mathrm{Fo} - \mathrm{Fo}^{(k+1)})\right\}; \quad \beta_k = H_k \frac{a^2}{h}.$$

The mathematical model (2) is a mixed problem of nonstationary heat conduction in which the presence of a "thermally thin" coating is allowed for by the generalized boundary condition at $\xi = 0$, which explicitly involves the time derivative of temperature. With account for the initial assumption of the form of the functional dependence $\beta = \beta(\mathrm{Fo})$, it can be ascertained [38] that in the standard class of functions there also exists a single solution of problem (2). In order to find this solution, by the linearity of the considered problem, we can use the known approach [36]: to assume that $\beta(\mathrm{Fo}) = \beta_0$ in problem (2) and find its solution $\theta_0(\xi, \mathrm{Fo})$, $\mathrm{Fo} \geq \mathrm{Fo}^{(0)} = 0$, which is sought when $0 \leq \mathrm{Fo} < \mathrm{Fo}^{(1)}$; to assume that $\beta(\mathrm{Fo}) \equiv \beta_1$ in problem (2) and with the initial condition $\theta_1(\xi, \mathrm{Fo}^{(1)}) = \theta_0(\xi, \mathrm{Fo}^{(1)})$ to find its solution $\theta_1(\xi, \mathrm{Fo})$, $\mathrm{Fo} \geq \mathrm{Fo}^{(1)}$, which is sought when $\mathrm{Fo}^{(1)} \leq \mathrm{Fo} < \mathrm{Fo}^{(2)}$, and so on. But due to the cumbersome form of the analytical expressions obtained in direct use of this approach and the laboriousness of numerical realization of them, in order attain the prime to goal of the investigations we used the following considerations.

Let

$$W(\xi, s) \overset{\Delta}{=} L\,[\theta\,(\xi, \mathrm{Fo})] \equiv \int_0^\infty \exp\,(-s\,\mathrm{Fo})\,\theta\,(\xi, \mathrm{Fo})\,d\,\mathrm{Fo} \tag{3}$$

be the integral Laplace transform [36] of the function $\theta(\xi, \mathrm{Fo})$ that, according to (2) and (3), satisfies the equation

$$sW(\xi, \mathrm{Fo}) = \frac{\partial^2 W(\xi, s)}{\partial \xi^2}\,, \quad \xi > 0\,, \tag{4}$$

the boundary condition

$$\gamma \frac{\partial W(\xi, s)}{\partial \xi} = L\,[\beta\,(\mathrm{Fo})\,\theta\,(\xi, \mathrm{Fo})] - L\,[\beta\,(\mathrm{Fo})] + sW(\xi, s)\,, \quad \xi = 0\,, \tag{5}$$

and for each fixed value of the parameter s belongs to the class of functions $L^2[0, \infty)$ integrated with the square in the spatial variable $\xi \in [0, \infty)$.

The solution $W(\xi, s)$ of Eq. (4) from the class $L^2[0, \infty)$ has the following form:

$$W(\xi, s) = V(s)\exp\,(-\xi\sqrt{s})\,, \tag{6}$$

where $V(s)$ is the integral Laplace transform of the function $\theta(0, \mathrm{Fo})$ setting the temperature distribution at the boundary $\xi = 0$, i.e.,

$$V(s) = W(0, s) \overset{\Delta}{=} L\,[\theta\,(0, \mathrm{Fo})]\,. \tag{7}$$

In this case, according to (5)–(7), the transform $V(s)$ must satisfy the relation

$$(s + \gamma\sqrt{s})\,V(s) + L\,[\beta\,(\mathrm{Fo})\,\theta\,(\xi, \mathrm{Fo})] = L\,[\beta\,(\mathrm{Fo})]\,, \tag{8}$$

i.e., with account for (6) we can ascertain that the initial problem will be solved in the integral Laplace transforms if we find solution of Eq. (8), where $\beta(\mathrm{Fo})$ is a known piecewise-continuous function.

Then let the function

$$\theta_k\,(\mathrm{Fo}) \overset{\Delta}{=} \theta\,(0, \mathrm{Fo})\left\{J\,(\mathrm{Fo} - \mathrm{Fo}^{(k)}) - J\,(\mathrm{Fo} - \mathrm{Fo}^{(k+1)})\right\} \tag{9}$$

specify the temperature at the boundary $\xi = 0$ for $\mathrm{Fo}^{(k)} \leq \mathrm{Fo} < \mathrm{Fo}^{(k+1)}$ and $k = \overline{0 : N}$, and

$$V_k(s) \overset{\Delta}{=} L\,[\theta_k\,\mathrm{Fo}] \equiv \int\limits_{\mathrm{Fo}^{(k)}}^{\mathrm{Fo}^{(k+1)}} \exp(-s\,\mathrm{Fo})\,\theta(0,\mathrm{Fo})\,d\,\mathrm{Fo} \qquad (10)$$

be its transforms. In this case, with account for (7), (9), and (10) we obtain

$$\theta(0,\mathrm{Fo}) = \sum_{k=0}^{N} \theta_k(\mathrm{Fo})\,; \quad V(s) = \sum_{k=0}^{N} V_k(s)\,. \qquad (11)$$

Moreover, according to (9)–(11) and [36], the equalities

$$\beta(\mathrm{Fo})\,\theta(0,\mathrm{Fo}) = \sum_{k=0}^{N} \beta_k\,\theta(0,\mathrm{Fo})\left\{J(\mathrm{Fo}-\mathrm{Fo}^{(k)}) - J(\mathrm{Fo}-\mathrm{Fo}^{(k+1)})\right\} = \sum_{k=0}^{N} \beta_k\,\theta_k(\mathrm{Fo})\,,$$

$$L\,[\beta(\mathrm{Fo})\,\theta(0,\mathrm{Fo})] = \sum_{k=0}^{N} \beta_k\,V_k(s)\,,$$

$$L\,[\beta(\mathrm{Fo})] = \sum_{k=0}^{N} \beta_k\,\frac{1}{s}\left\{\exp(-s\,\mathrm{Fo}^{(k)}) - \exp(-s\,\mathrm{Fo}^{(k+1)})\right\},$$

occur, with account for which relation (8) can be represented in the form of the equation with $N+1$ unknowns $\{V_k(s)\}_{k=0}^{N}$:

$$\sum_{k=0}^{N} \left\{(s+\gamma\sqrt{s}+\beta_k)\,V_k(s) - \beta_k\,\frac{1}{s}\,[\exp(-s\,\mathrm{Fo}^{(k)}) - \exp(-s\,\mathrm{Fo}^{(k+1)})]\right\} = 0\,. \qquad (12)$$

In this case, according to (10), the unknowns $\{V_k(s)\}_{k=0}^{N}$ which enter Eq. (12) are not independent and the principal scheme of their relations has the following form: $V_0(s) \Rightarrow V_1(s) \Rightarrow \ldots \Rightarrow V_N(s)$. Therefore, to find the transforms $\{V_k(s)\}_{k=0}^{N}$, i.e., to solve Eq. (12), we use the successive procedure. We also note that the initial assumption on the "thermal thickness" of the coating corresponds to the presence of two real roots

$$\mu_{1k} = 0.5\,(\gamma + \sqrt{\gamma^2 - 4\beta_k})\,, \quad \mu_{2k} = 0.5\,(\gamma - \sqrt{\gamma^2 - 4\beta_k}) \qquad (13)$$

for the equation

$$\mu^2 + \gamma\mu + \beta_k = 0\,, \quad k = \overline{0:N}\,.$$

Thus,

$$\frac{1}{s+\gamma\sqrt{s}+\beta_k} = \frac{1}{\sqrt{\gamma^2-4\beta_k}}\left(\frac{1}{\sqrt{s}+\mu_{2k}} - \frac{1}{\sqrt{s}+\mu_{1k}}\right), \qquad (14)$$

where the real constants μ_{1k} and μ_{2k} are determined by equalities (13) for each $k = \overline{0:N}$.

In the first stage of realization of the successive procedure of solution of Eq. (12), we set $N = 0$, which corresponds to $\mathrm{Fo}^{(0)} = \infty$ and $\beta(\mathrm{Fo}) \equiv \beta_0$. Having denoted the solution of Eq. (12) by $V_0^{\infty}(s)$ (with the assumption made) and its inverse transform by $\theta_0^\infty(\mathrm{Fo})$, we arrive at the following equation for determination of the transforms $V_0^\infty(s)$:

$$(s + \gamma\sqrt{s} + \beta_0)\, V_0^\infty(s) - s^{-1}\beta_0 = 0 .$$

Consequently, with account for (14) and (13) we have

$$V_0^\infty(s) = \frac{\beta_0}{s\,(s + \gamma\sqrt{s} + \beta_0)} = \frac{\beta_0}{\sqrt{\gamma^2 - 4\beta_0}}\left[\frac{1}{s\,(\sqrt{s} + \mu_{20})} - \frac{1}{s\,(\sqrt{s} + \mu_{10})}\right]. \quad (15)$$

Having applied the L^{-1}-operator of the inverse integral Laplace transform to the right- and left-hand sides of equality (15) [36], we find the function

$$\theta_0^\infty(\mathrm{Fo}) = \frac{\beta_0}{\sqrt{\gamma^2 - 4\beta_0}}\left\{\frac{1}{\mu_{20}}\left[1 - \exp(\mu_{20}^2\,\mathrm{Fo})\,\mathrm{erfc}(\mu_{20}\sqrt{\mathrm{Fo}})\right] - \right.$$

$$\left. - \frac{1}{\mu_{10}}\left[1 - \exp(\mu_{10}^2\,\mathrm{Fo})\,\mathrm{erfc}(\mu_{10}\sqrt{\mathrm{Fo}})\right]\right\}, \quad \mathrm{Fo} \ge \mathrm{Fo}^{(0)} = 0 , \quad (16)$$

where $\mathrm{ercf}\{u\} = \frac{2}{\sqrt{\pi}}\int_u^\infty \exp(-z^2)dz$ is the additional Gauss error function which for $0 = \mathrm{Fo}^{(0)} \le \mathrm{Fo} < \mathrm{Fo}^{(1)}$ coincides with the solution $\theta_0(\mathrm{Fo})$ of problem (2) at $\xi = 0$, i.e.,

$$\theta_0(\mathrm{Fo}) = \theta_0^\infty(\mathrm{Fo})\left\{J(\mathrm{Fo} - \mathrm{Fo}^{(0)}) - J(\mathrm{Fo} - \mathrm{Fo}^{(1)})\right\} . \quad (17)$$

Thus, the solution of problem (2) at $\xi = 0$ and $0 \le \mathrm{Fo} < \mathrm{Fo}^{(1)}$ is fully determined by equalities (17), (16), and (13). Moreover, according to (17), there occurs the equality

$$V_0(s) = V_0^\infty(s) - \Psi_0(s) , \quad \Psi_0(s) \overset{\Delta}{=} L\left[\theta_0^\infty(\mathrm{Fo})\, J(\mathrm{Fo} - \mathrm{Fo}^{(1)})\right] , \quad (18)$$

where the transform $\Psi_0(s)$ with account for (16) can be represented in explicit form, but its further use lacks any prospects. We also note that equalities (18) and (15) allow transformation of Eq. (12) to the following:

$$\sum_{k=1}^{N}\left\{(s + \gamma\sqrt{s} + \beta_k)\, V_k(s) - \beta_k \frac{1}{s}\left[\exp(-s\,\mathrm{Fo}^{(k)}) - \exp(-s\,\mathrm{Fo}^{(k+1)})\right]\right\} =$$

$$= (s + \gamma\sqrt{s} + \beta_0)\,\Psi_0(s) - s^{-1}\beta_0 \exp(-s\,\mathrm{Fo}^{(1)}) . \quad (19)$$

In the second stage we set $N = 1$, which corresponds to $\mathrm{Fo}^{(2)} = \infty$ and $\beta(\mathrm{Fo}) = \beta_0\{1 - J(\mathrm{Fo} - \mathrm{Fo}^{(1)})\} + \beta_1 J(\mathrm{Fo} - \mathrm{Fo}^{(1)})$. Having denoted the solution of Eq. (19) by $V_1^\infty(s)$ (with the assumption made) and its inverse transform by $\theta_1^\infty(\mathrm{Fo})$, we arrive at the following equation for determining the transform $V_1^\infty(s)$:

$$(s + \gamma\sqrt{s} + \beta_1)\, V_1^\infty(s) - s^{-1}\beta_1 \exp(-s\,\mathrm{Fo}^{(1)}) = (s + \gamma\sqrt{s} + \beta_0)\,\Psi_0(s) - s^{-1}\beta_0 \exp(-s\,\mathrm{Fo}^{(1)}) ,$$

from which we find

$$V_1^\infty(s) = \frac{\beta_1 - \beta_0}{s(s + \gamma\sqrt{s} + \beta_1)} \exp(-s\,\mathrm{Fo}^{(1)}) + \left(1 - \frac{\beta_1 - \beta_0}{s + \gamma\sqrt{s} + \beta_1}\right)\Psi_0(s)\,. \tag{20}$$

Having applied the L^{-1}-operator of the inverse integral Laplace transform to the right- and left-hand sides of the obtained equality with account for (14) and (18) and having used the convolution theorem [36], we find the function

$$\theta_1^\infty(\mathrm{Fo}) = \frac{\beta_1 - \beta_0}{\sqrt{\gamma^2 - 4\beta_1}} \left\{\frac{1}{\mu_{21}}[1 - \exp[\mu_{21}^2(\mathrm{Fo} - \mathrm{Fo}^{(1)})]\,\mathrm{erfc}(\mu_{21}\sqrt{\mathrm{Fo} - \mathrm{Fo}^{(1)}})] - \right.$$

$$\left. - \frac{1}{\mu_{11}}[1 - \exp[\mu_{11}^2(\mathrm{Fo} - \mathrm{Fo}^{(1)})]\,\mathrm{erfc}(\mu_{11}\sqrt{\mathrm{Fo} - \mathrm{Fo}^{(1)}})]\right\} + \theta_0^\infty(\mathrm{Fo}) -$$

$$- \frac{\beta_1 - \beta_0}{\sqrt{\gamma^2 - 4\beta_1}} \int\limits_{\mathrm{Fo}^{(1)}}^{\mathrm{Fo}} [\mu_{11}\exp[\mu_{11}^2(\mathrm{Fo} - \tau)]\,\mathrm{erfc}(\mu_{11}\sqrt{\mathrm{Fo} - \tau}) -$$

$$- \mu_{21}\exp[\mu_{21}^2(\mathrm{Fo} - \tau)]\,\mathrm{erfc}(\mu_{21}\sqrt{\mathrm{Fo} - \tau})]\,\theta_0^\infty(\tau)\,d\tau\,, \quad \mathrm{Fo} \ge \mathrm{Fo}^{(1)}\,, \tag{21}$$

which for $\mathrm{Fo}^{(1)} \le \mathrm{Fo} \le \mathrm{Fo}^{(2)}$ coincides with the solution of problem (2) at $\xi = 0$, i.e.,

$$\theta_1(\mathrm{Fo}) = \theta_1^\infty(\mathrm{Fo})\left\{J(\mathrm{Fo} - \mathrm{Fo}^{(1)}) - J(\mathrm{Fo} - \mathrm{Fo}^{(2)})\right\}. \tag{22}$$

In this case, the function $\theta_0^\infty(\mathrm{Fo})$ and the values of the quantities μ_{11} and μ_{21} are determined according to (16) and (13), respectively.

In the integral Laplace transforms, we represent equality (22) as

$$V_1(s) = V_1^\infty(s) - \Psi_1(s)\,, \quad \Psi_1(s) = L[\theta_1^\infty(\mathrm{Fo})\,J(\mathrm{Fo} - \mathrm{Fo}^{(2)})]\,, \tag{23}$$

where the transform $\Psi_1(s)$ is fully determined according to (21). Moreover, according to (23) and (20), we transform Eq. (19) to the following:

$$\sum_{k=2}^{N}\left\{(s + \gamma\sqrt{s} + \beta_k)\,V_k(s) - \beta_k\frac{1}{s}[\exp(-s\,\mathrm{Fo}^{(k)}) - \exp(-s\,\mathrm{Fo}^{(k+1)})]\right\} =$$

$$= (s + \gamma\sqrt{s} + \beta_1)\,\Psi_1(s) - s^{-1}\beta_1\exp(-s\mathrm{Fo}^{(2)})\,. \tag{24}$$

Having compared Eqs. (19) and (24), we arrive at the following conclusion: if the index k takes successively values 2, 3, ..., N, then

$$\theta_k(\mathrm{Fo}) = \theta_k^\infty(\mathrm{Fo})\left\{J(\mathrm{Fo} - \mathrm{Fo}^{(k)}) - J(\mathrm{Fo} - \mathrm{Fo}^{(k+1)})\right\},$$

$$\theta_k^\infty(\mathrm{Fo}) = \frac{\beta_k - \beta_{k-1}}{\sqrt{\gamma^2 - 4\beta_k}}\left\{\frac{1}{\mu_{2k}}[1 - \exp[\mu_{2k}^2(\mathrm{Fo} - \mathrm{Fo}^{(k)})]\,\mathrm{erfc}(\mu_{2k}\sqrt{\mathrm{Fo} - \mathrm{Fo}^{(k)}})] - \right.$$

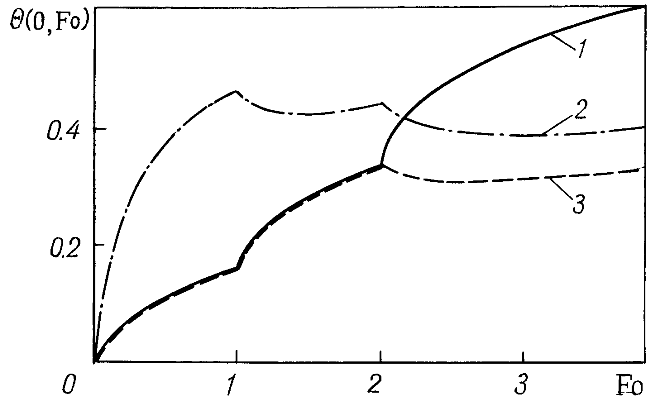

Fig. 1. Dependence of the temperature θ(0, Fo) of the boundary $\xi = 0$ on the Fourier number Fo in the pulsed mode of heat exchange with the environment for $N = 2$, $\mathrm{Fo}^{(1)} = 1$, $\mathrm{Fo}^{(2)} = 2$, $\gamma = 5$, and different values of the parameters β_0, β_1, and β_2: 1) $\beta_0 = 1$, $\beta_1 = 2$, and $\beta_2 = 4$; 2) 4, 2, and 1; 3) 1, 2, and 1.

$$-\frac{1}{\mu_{1k}}\left[1-\exp\left[\mu_{1k}^2\left(\mathrm{Fo}-\mathrm{Fo}^{(k)}\right)\right]\mathrm{erfc}\left(\mu_{1k}\sqrt{\mathrm{Fo}-\mathrm{Fo}^{(k)}}\right)\right]\Bigg\}+\theta_{k-1}^{\infty}(\mathrm{Fo})-$$

$$-\frac{\beta_k-\beta_{k-1}}{\sqrt{\gamma^2-4\beta_k}}\int_{\mathrm{Fo}^{(k)}}^{\mathrm{Fo}}\left[\mu_{1k}\exp\left[\mu_{1k}^2(\mathrm{Fo}-\tau)\right]\mathrm{erfc}\left(\mu_{1k}\sqrt{\mathrm{Fo}-\tau}\right)-\right.$$

$$\left.-\mu_{2k}\exp\left[\mu_{2k}^2(\mathrm{Fo}-\tau)\right]\mathrm{erfc}\left(\mu_{2k}\sqrt{\mathrm{Fo}-\tau}\right)\right]\theta_{k-1}^{\infty}(\tau)\,d\tau\,,\quad \mathrm{Fo}\geq\mathrm{Fo}^{(k)}\,,$$

where for $k = 2$ the function $\theta_1^{\infty}(\mathrm{Fo})$ is determined by equality (21) and the values of the quantities μ_{1k} and μ_{2k} are found by (13). Since equality (11) occurs, the initial problem of determination of the temperature θ(0, Fo) at the boundary of a solid body modeled by a half-space with a "thermally thin" coating in realization of the pulsed modes of heat exchange with the environment is fully solved.

We consider some results of the investigations with that reflect the most characteristic features of the process of formation of the temperature profile θ(0, Fo) at the boundary of a half-space with a "thermally thin" coating in realization of the pulsed modes of heat exchange with the environment.

Figure 1 presents the dependence of the temperature θ(0, Fo) of the half-space boundary on the Fourier number in the pulsed mode of heat exchange with the environment characterized by the functional dependence

$$\beta(\mathrm{Fo})=\beta_0\left\{1-J(\mathrm{Fo}-1)\right\}+\beta_1\left\{J(\mathrm{Fo}-1)-J(\mathrm{Fo}-2)\right\}+\beta_2\,J(\mathrm{Fo}-2)$$

for different values of the parameters β_0, β_1, and β_2. In this case, the inequality $\beta_i > \beta_{i+1}$ corresponds to deterioration of the conditions of heat exchange with the environment on the $(i+1)$th time interval, and the inequality $\beta_i < \beta_{i+1}$ corresponds to improvement of them.

The improvement of the heat-exchange conditions is accompanied by a sharp increase in the temperature at the half-space boundary, especially in the initial stage (see Fig. 1, curve 1 for $0 \leq \mathrm{Fo} < \infty$ and curve 3 for $0 \leq \mathrm{Fo} < 2$). The deterioration of the heat-exchange conditions leads to the formation of the characteristic relaxation zone (see Fig. 1, curve 3 for $2 \leq \mathrm{Fo} < \infty$ and curve 2 for $0 \leq \mathrm{Fo} < \infty$). In this case, the duration of the period of a monotonic decrease in the temperature in this zone is determined by both the quantity

$(\beta_i - \beta_{i+1})$ and the duration $(\mathrm{Fo}^{(i+1)} - \mathrm{Fo}^{(i)})$ of the previous phase. But in realization of any pulsed mode of heat exchange with the environment determined by the functional dependence

$$\beta(\mathrm{Fo}) = \sum_{k=0}^{N} \beta_k \left\{ J(\mathrm{Fo} - \mathrm{Fo}^{(k)}) - J(\mathrm{Fo} - \mathrm{Fo}^{(k+1)}) \right\},$$

where $\mathrm{Fo}^{(0)} = 0$ and $\mathrm{Fo}^{(N+1)} = \infty$, for the temperature $\theta(0, \mathrm{Fo})$ at the half-space boundary the asymptotic estimate (for large Fo values)

$$\theta(0, \mathrm{Fo}) \sim 1 - \frac{1}{\beta_N \sqrt{\pi \mathrm{Fo}}},$$

occurs, i.e., the qualitative character of the behavior of the function $\theta(0, \mathrm{Fo})$ for $\mathrm{Fo} \to \infty$ does not depend on the realized modes of heat exchange with the environment: $\theta(0, \mathrm{Fo}) \to 1$ when $\mathrm{Fo} \to \infty$.

NOTATION

x, spatial variable; t, time; T, temperature; $\xi = x/x_*$, dimensionless variable; $\mathrm{Fo} = \kappa t/x_*^2$, Fourier number; $\mathrm{Bi} = \alpha x_*/\lambda_{\mathrm{coat}}$, Biot number; x_*, chosen scale unit; l, coating thickness; λ, thermal conductivity; κ, thermal diffusivity; $\alpha = \alpha(t)$, heat-transfer coefficient. Subscripts: coat, coating; env, environment.

REFERENCES

1. K. A. Kiselev and A. I. Lazarev, *Zh. Tekh. Fiz.*, **30**, Issue 6, 28–33 (1960).
2. Yu. L. Rozenshtok, *Inzh.-Fiz. Zh.*, No. 3, 45–50 (1963).
3. N. M. Prikhod'ko, *Izv. Vyssh. Uchebn. Zaved., Aviats. Tekh.*, No. 3, 21–27 (1963).
4. L. B. Bodretsova, *Mat. Sb.*, **64**, No. 2, 223–233 (1964).
5. M. M. Sidlyar, *Prikl. Mekh.*, **1**, Issue 7, 11–13 (1965).
6. V. F. Vorob'ev, *Prikl. Mekh.*, **1**, Issue 7, 14–20 (1965).
7. V. V. Ivanov and V. V. Salomatov, *Inzh.-Fiz. Zh.*, **9**, No. 1, 83–85 (1965).
8. V. V. Ivanov and V. V. Salomatov, *Inzh.-Fiz. Zh.*, **11**, No. 2, 266–268 (1966).
9. Yu. V. Vidin, *Inzh.-Fiz. Zh.*, **11**, No. 2, 166–170 (1966).
10. Yu. V. Vidin, *Izv. Akad. Nauk SSSR, É nergetika Transport*, No. 3, 110–114 (1967).
11. Yu. V. Vidin, *Izv. Akad. Nauk SSSR, É nergetika Transport*, No. 4, 132–134 (1967).
12. V. V. Salomatov and É. I. Goncharov, *Izv. Akad. Nauk SSSR, É nergetika Transport*, No. 4, 99–103 (1967).
13. V. V. Salomatov and É. I. Goncharov, *Inzh.-Fiz. Zh.*, **14**, No. 4, 743–745 (1968).
14. V. V. Salomatov, V. V. Ivanov, and É. I. Goncharov, *Izv. Akad. Nauk SSSR, É nergetika Transport*, No. 4, 144–147 (1968).
15. V. V. Salomatov and É. I. Goncharov, *Izv. Akad. Nauk SSSR, É nergetika Transport*, No. 6, 154–159 (1969).
16. B. Ya. Lyubov and N. I. Yalovoi, *Inzh.-Fiz. Zh.*, **17**, No. 4, 679–687 (1969).
17. V. N. Kozlov, *Inzh.-Fiz. Zh.*, **18**, No. 1, 133–138 (1970).
18. Yu. S. Postol'nik, *Inzh.-Fiz. Zh.*, **18**, No. 2, 316–322 (1970).
19. I. M. Prikhod'ko, *Inzh.-Fiz. Zh.*, **18**, No. 2, 323–327 (1970).
20. V. V. Salomatov, *Izv. Akad. Nauk SSSR, É nergetika Transport*, No. 3, 165–170 (1970).
21. É. M. Kartashov and V. E. Mos'yakov, *Izv. Vyssh. Uchebn. Zaved., Aviats. Tekh.*, No. 1, 75–82 (1973).

22. P. V. Tsoi, *Izv. Akad. Nauk SSSR, Énergetika Transport*, No. 4, 117-126 (1976).
23. Ya. S. Podstrigach, Yu. M. Kolyano, V. I. Gromovyk, and V. L. Lyuzben', *Thermal Elasticity of Bodies for Variable Coefficients of Heat Transfer* [in Russian], Kiev (1977).
24. Yu. S. Postol'nik, *Approximate Methods of Investigations in Thermomechanics* [in Russian], Kiev–Donetsk (1984).
25. É. M. Kartashov, *Analytical Methods in the Theory of Heat Conduction of Solids* [in Russian], Moscow (1985).
26. A. V. Attetkov and I. K. Volkov, *Vestn. MGU, Mashinostroenie*, No. 1, 49–56 (1999).
27. A. V. Attetkov, L. N. Vlasova, I. K. Volkov, and E. A. Zagoruiko, *Vestn. MGU, Mashinostroenie*, No. 2, 8–10 (1999).
28. Yu. V. Vidin, in: *Proc. 2nd Russ. Nat. Conf. on Heat Transfer*, [in Russian], Vol. 7, Moscow (1998), pp. 49–50.
29. Yu. V. Vidin, in: *Proc. IV Minsk Int. Forum "Heat and Mass Transfer–MIF-2000,"* May 22–26, 2000, Minsk, Vol. 3 [in Russian], Minsk (2000), pp. 386–388.
30. A. V. Attetkov and I. K. Volkov, in: *Proc. IV Minsk Int. Forum "Heat and Mass Transfer–MIF-2000,"* May 22–26, 2000, Minsk, Vol. 3 [in Russian], Minsk (2000), pp. 370–382.
31. A. V. Attetkov and I. K. Volkov, *Vestn. MGU, Mashinostroenie*, No. 4, 3–10 (1999).
32. V. S. Zarubin, *Temperature Fields in the Structure of Aircraft* [in Russian], Moscow (1978).
33. B. M. Pankratov, Yu. V. Polezhaev, and A. K. Rud'ko, *Interaction of Materials with Gas Flows* [in Russian], Moscow (1976).
34. Yu. V. Polezhaev and F. B. Yurevich, *Heat Protection* [in Russian], Moscow (1976).
35. V. S. Zarubin, *Calculation and Optimization of Thermal Insulation* [in Russian], Moscow (1991).
36. A. V. Luikov, *Theory of Heat Conduction* [in Russian], Moscow (1967).
37. G. M. Bartenev and A. I. Zhornik, *Fiz. Khim. Obrab. Mater.*, No. 3, 100–108 (1972).
38. O. A. Ladyzhenskaya, V. A. Solonnikov, and N. N. Ural'tseva, *Linear and Quasilinear Parabolic Equations* [in Russian], Moscow (1967).